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2009 J. Phys.: Condens. Matter 21 275901

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# Negative pressure induced ferroelectric phase transition in rutile TiO<sub>2</sub>

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Received 9 December 2008, in final form 24 May 2009

Published 12 June 2009

Online at [stacks.iop.org/JPhysCM/21/275901](http://stacks.iop.org/JPhysCM/21/275901)

## Abstract

First-principles pseudopotential calculations by means of the local density approximation (LDA) within density-functional theory (DFT) are carried out to investigate the negative pressure induced ferroelectric phase transition in rutile TiO<sub>2</sub> in the range of  $-25$  to  $25$  GPa. The softening behavior of the A<sub>2u</sub>(TO) modes at the  $\Gamma$  point following the decreasing pressure leads to a ferroelectric phase transition from  $P4_2/mnm$  (rutile) space group to  $P42nm$  (*ferro*) space group. The calculated pressure dependence of the phonon frequencies, A<sub>1</sub>(TO), E(TO), and B<sub>2</sub> modes of the *ferro* TiO<sub>2</sub> relative to A<sub>2u</sub>(TO), E<sub>u</sub>(TO), and B<sub>1u</sub> modes of rutile TiO<sub>2</sub>, indicates that the phase transition occurs at around  $-10$  GPa, consistent with the total energy results. The dramatic increase of the *c*-axial dielectric tensor in the vicinity of the phase transition indicates it to be a typical ferroelectric phase transition. The character of the phase transition is generally identified in terms of the calculated order parameters, displaying a second order.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

Titanium dioxide (TiO<sub>2</sub>) is of great importance in both fundamental research and technical applications, exhibiting high photon-to-electron conversion efficiencies [1], reactivity photocatalysis [2], and large dielectric constants [3]. Besides naturally existent polymorphic forms, including rutile, anatase, and brookite phases, a series of new high pressure structured TiO<sub>2</sub> with interesting properties is obtained at elevated pressures. For instance, a high pressure phase TiO<sub>2</sub> with cotunnite structure (*c*-TiO<sub>2</sub>) appears as the hardest oxide ever known, which is synthesized under pressures over 60 GPa and temperatures over 1000 K [4].

A large amount of experimental and theoretical work has been done, focusing on the high pressure induced phase transition of TiO<sub>2</sub>. The transformations from the anatase to the columbite and baddeleyite phases have been detected under 2.6–8 GPa and 13–17 GPa respectively with Raman spectroscopy and x-ray diffraction [5, 6]. Using first-

principles calculations, Sasaki [7] has indicated that the rutile–columbite–baddeleyite phase transition occurs energetically at 7.5 and 26 GPa, respectively, while Muscat *et al* [8] have shown that the computed anatase–columbite, rutile–columbite, columbite–baddeleyite, and baddeleyite–cotunnite phase transitions appear energetically at about 3.5, 21, 31, and 63 GPa, respectively. These results give a very good insight into the high pressure induced phase transition linking these TiO<sub>2</sub> polymorphs together.

Rutile TiO<sub>2</sub> is also classified to be an incipient ferroelectric, which behaves as a ferroelectric material in its paraelectric phase and shows a rapid increase of the dielectric constant along the *c* axis following the decrease of temperature. However, even when the temperature reaches 0 K the ferroelectric transition does not occur [9]. Montanari *et al* [10] performed a plane-wave pseudopotential implementation of the density function within the framework of the local density approximation on rutile TiO<sub>2</sub>, showing that under negative pressure or uniaxial strain a ferroelectric phase transition will occur. However, the sequential TiO<sub>2</sub> ferroelectric phase has not been characterized.

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**Table 1.** Fully relaxed structural parameters for rutile and ferro TiO<sub>2</sub>.

	<i>V</i>	<i>a</i>	<i>c</i>	<i>u</i>	<i>v</i> (×10 <sup>4</sup> )
Rutile (present)	59.9614	4.5344	2.9163	0.303 788	
Rutile (calc.) [10]	60.4779	4.551	2.920	0.3040	
Rutile (calc.) [14]	61.1887	4.5672	2.9334	0.3038	
Rutile (exp.) [15]	62.3616	4.59	2.96	0.306	
Ferro (present)	59.9673	4.5347	2.9162	0.303 758	6.1554

In the present paper, we focus on the negative pressure induced ferroelectric phase transition in rutile TiO<sub>2</sub>. The density-functional theory (DFT) calculations within the local density approximation (LDA) were performed to study the pressure dependence of the TiO<sub>2</sub> structure and the corresponding properties. The TiO<sub>2</sub> ferroelectric phase was characterized. We also identified the character of the ferroelectric phase transition. In practice, the application of negative hydrostatic pressure is challenging. However, it has been shown that in-plane tensile stress occurring in epitaxial films is not only able to adjust Curie temperature but also able to change an incipient ferroelectric, for instance SrTiO<sub>3</sub>, to a true ferroelectric [23]. Thus the followed theoretical studies about the effect of negative pressures are very useful for better understanding of the structural instability of TiO<sub>2</sub> and possibly provide an avenue to obtain ferroelectrics from incipient materials.

## 2. Method

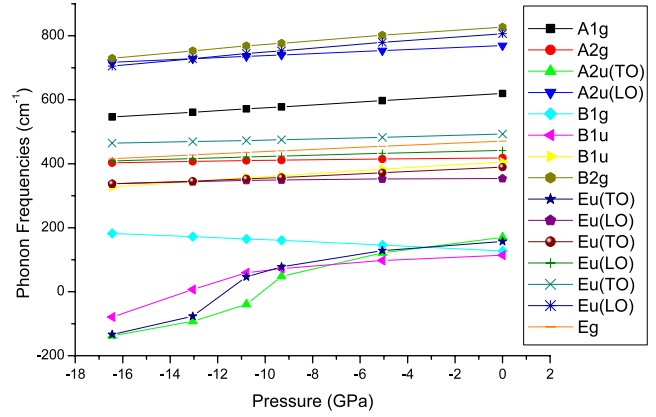
A first-principles density-functional theory (DFT) within the local density approximation (LDA) was employed to perform the theoretical investigation. The extended norm-conserving pseudopotentials including more states as semi-core than in the Troullier–Martins’ were used for all atoms [11], considering Ti(3s, 3p, 3d, 4s) and O(2s, 2p) orbits expended using plane waves with a cutoff energy of 55 Hartree. A 6 × 6 × 8 Monkhorst–Pack mesh was employed to perform the integration in the reciprocal space. These parameters were tested and showed enough convergence for the energy calculation. The convergence of the structural optimizations requires the Hellmann–Feynman forces to be less than 10<sup>-6</sup> hartree/bohr (≈5 × 10<sup>-5</sup> eV Å<sup>-1</sup>). The phonon frequencies and the dielectric tensors of TiO<sub>2</sub> were calculated based on the density-functional perturbation theory (DFPT) [12].

All of these simulations are performed by using the *ABINIT* software package [13] at cluster *Lenovo 1800*.

## 3. Results

### 3.1. Isotropic pressure dependence of structural stability for rutile TiO<sub>2</sub>

The fully optimized structure parameters for rutile TiO<sub>2</sub> are listed in table 1. The theoretical lattice constants are smaller than the experimental values due to the LDA’s character, but the differences are less than 1.5%. The resident pressures



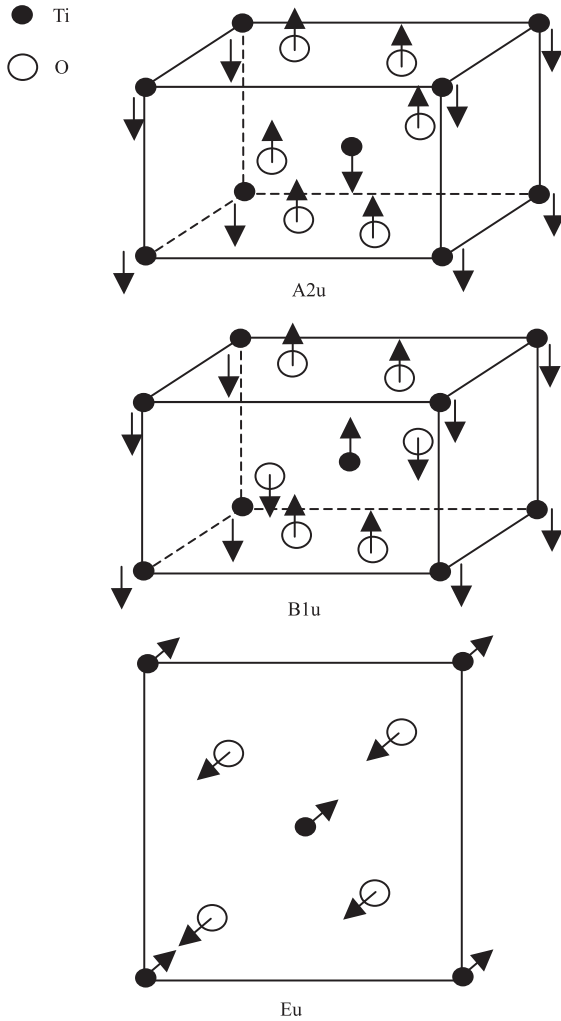
**Figure 1.** Calculated phonon frequencies at the  $\Gamma$  point of rutile TiO<sub>2</sub> as a function of pressure.

**Table 2.** Calculated phonon frequencies at  $\Gamma$  point for rutile and ferro TiO<sub>2</sub> with fully relaxed structural parameters.

Rutile	Rutile (present)	Rutile (calc.) [18]	Rutile (exp.) [20, 21]	Ferro
B <sub>1u</sub>	113.9774	116.7	113 [19]	B <sub>2</sub> 113.8058
B <sub>1g</sub>	127.0661	125.2	143	B <sub>1</sub> 127.2529
E <sub>u</sub> (TO)	157.0934	164.8	183	E(TO) 157.1486
A <sub>2u</sub> (TO)	169.8428	176.1	167	A <sub>1</sub> (TO) 169.3745
E <sub>u</sub> (LO)	353.5886	351.5	373	E(LO) 353.6248
E <sub>u</sub> (TO)	389.5581	391.3	388	E(TO) 389.5850
B <sub>1u</sub>	405.5413	407.5	406 [19]	B <sub>2</sub> 405.2915
A <sub>2g</sub>	417.917	415.5		A <sub>2</sub> 417.9510
E <sub>u</sub> (LO)	441.4259	441.7	458	E(LO) 441.4318
E <sub>g</sub>	470.9252	471.5	447	E(TO) 470.6107
				E(LO) 470.7439
E <sub>u</sub> (TO)	492.8863	492.8	500	E(TO) 492.9528
A <sub>1g</sub>	619.3465	622.5	612	A <sub>1</sub> (TO) 619.0701
				A <sub>1</sub> (LO) 619.0888
A <sub>2u</sub> (LO)	769.2788	769.3	812	A <sub>1</sub> (LO) 769.0187
E <sub>u</sub> (TO)	806.4344	808.4	807	E(LO) 806.4271
B <sub>2g</sub>	827.0474	828.0	827	B <sub>2</sub> 826.9440

of the unit cell with optimized lattice constants are less than 1 × 10<sup>-4</sup> GPa, which is sufficiently small for the practical calculations related to the phonon properties. The calculated phonon frequencies at the  $\Gamma$  point of rutile TiO<sub>2</sub> are shown in table 2. The experimental neutron scattering [19], infrared and Raman [20, 21] and other calculated results are also listed. The present results have a good agreement with the experimental and theoretical values; the deviation from other calculations is less than ~10 cm<sup>-1</sup>.

Starting from the fully relaxed structure, the phonon frequencies at the  $\Gamma$  point are calculated as a function of negative pressure as shown in figure 1. Except for the B<sub>1g</sub> mode, all the vibration modes become softer as pressure decreases. The B<sub>2g</sub>, E<sub>u</sub>(LO), and A<sub>2u</sub>(LO) are the three stiffest modes, while the A<sub>2u</sub>(TO), E<sub>u</sub>(TO), and B<sub>1u</sub> are the three softest modes and become negative values at about -10 GPa, -11.5 GPa, and -13 GPa, respectively. These trends have a good agreement with Montanari’s results [10], which, however, predict that the instability occurs at around -4 GPa. The present calculations confirm the previous prediction that rutile TiO<sub>2</sub> becomes unstable with the softening of the A<sub>2u</sub> modes.



**Figure 2.** Schematic image of atomic displacements associated with the eigenvector corresponding to the  $A_{2u}$ (TO),  $E_u$ (TO), and  $B_{1u}$  modes.

The atomic displacements associated with the  $A_{2u}$ (TO),  $B_{1u}$ , and  $E_u$ (TO) modes are schematically shown in figure 2. The softest  $A_{2u}$  mode involves a rigid displacement of the Ti sublattice along the  $c$ -axis and an analogous displacement of the O sublattice but with an opposite sign. This displacement will change  $TiO_2$  from the  $P4_2/mnm$  ( $D_{4h}$ ) space group to the  $P4_2nm$  ( $C_{4v}$ ) space group. The atomic displacements associated with the  $B_{1u}$  modes involve relative displacements of apical Ti and center Ti atoms and those of external and internal O atoms, changing rutile  $TiO_2$  into the  $P\bar{4}21m$  ( $D_{2D}^3$ ) space group. The atomic displacements with the  $E_u$  modes involve similar displacements with the  $A_{2u}$ (TO) modes, except they move along the  $\langle 110 \rangle$  direction, changing rutile  $TiO_2$  into the  $Pm$  ( $C_s$ ) space group. The sequential phases corresponding to the  $A_{2u}$  and  $E_u$  modes are possibly ferroelectric while the sequential phase corresponding to  $B_{1u}$  is paraelectric. In this paper, we only pay attention to the ferroelectric phase of  $TiO_2$  related to the  $A_{2u}$ (TO) modes, referring to it as *ferro* phase in the following sections.

### 3.2. Pressure induced phase transition from rutile to *ferro* $TiO_2$

The *ferro*  $TiO_2$  is characterized by the tetragonal space group  $P4_2nm$ . The unit cell contains two  $TiO_2$  units with Ti ions at 2a Wyckoff positions  $(0, 0, v)$  and  $(1/2, 1/2, v + 1/2)$  and O ions at 4c Wyckoff positions  $(u, u, z)$ ,  $(-u, -u, v)$ ,  $(-u + 1/2, u + 1/2, v + 1/2)$ ,  $(u + 1/2, -u + 1/2, v + 1/2)$ . Fully optimized structural parameters and corresponding phonon frequencies are calculated, as shown in tables 1 and 2, respectively. It is found that the lattice constants of the *ferro*  $TiO_2$  are very close to the values of rutile  $TiO_2$ , as well as the phonon frequencies. In fact, the difference of the corresponding phonon frequencies shown in table 2 is  $\sim 1 \text{ cm}^{-1}$ . Note that, from the factor group analysis for the *ferro* primitive cell, the optical modes at the  $\Gamma$  point are classified into the following symmetry species:

$$\Gamma_{opt} = 2A_1 + A_2 + B_1 + 3B_2 + 4E. \quad (1)$$

The cohesive energies as a function of unit cell volume corresponding to restrict relaxed structural parameters (fixing the unit cell volume) are calculated as shown in figure 3(a), and the volume expandabilities as a function of pressure for rutile and *ferro*  $TiO_2$  are shown in figure 3(b). The bulk modulus of rutile and *ferro*  $TiO_2$  are predicted from the curves to be  $B = 250.2$  and  $243.9$  GPa, in agreement with the experimental values of  $230 \pm 20$  GPa [16, 17]. However, under negative pressure the *ferro*  $TiO_2$  is more energetically stable and compressible than rutile  $TiO_2$ , consistent with the fact that the ferroelectricity favors the more compressible structure and larger volume. Using the curves in figure 3, the rutile-to-*ferro* transition pressure at  $T = 0$  K can be predicted by the enthalpy difference with the following equation:

$$\Delta H_{f-r} = \Delta E_{f-r} + \Delta V_{f-r} \times P \quad (2)$$

where  $H$  and  $E$  are the enthalpy and total energy of the unit cell; the subscripts  $f$  and  $r$  refer to the *ferro* and rutile  $TiO_2$  respectively. Because of the very small enthalpy difference between the *ferro* and rutile as shown in figure 4, the calculated phase transition pressure needs to be examined by directly studying energy derivative properties, such as phonon frequencies and dielectric constants. The calculated pressure dependences of the phonon frequencies at the  $\Gamma$  point and the dielectric tensor along the  $c$  direction in the *ferro*  $TiO_2$  are shown in figures 5 and 6, respectively. It can be found that the three lowest  $A_1$ (TO),  $E$ (TO), and  $B_2$  modes, which are related to the  $A_{2u}$ (TO),  $E_u$ (TO), and  $B_{1u}$  modes in rutile  $TiO_2$  respectively, exhibit softening behaviors until the pressure decreases to  $-10$  GPa, and then become stiffer as the pressure decreases further. Other phonon modes, such as  $E$ (LO) at  $441 \text{ cm}^{-1}$ ,  $E$ (TO) at  $470 \text{ cm}^{-1}$ ,  $E$ (TO) at  $492 \text{ cm}^{-1}$ , and  $A_1$ (LO) at  $619 \text{ cm}^{-1}$ , also show abrupt changes at around  $-10$  GPa. The calculated  $c$ -axis dielectric tensor shows a dramatic increase at around  $-10$  GPa, more than sixfold compared with the values at zero pressure, implying that this process is a typical ferroelectric phase transition. The estimated phase transition pressure,  $-10$  GPa, agrees well with the total energy results (figure 4).

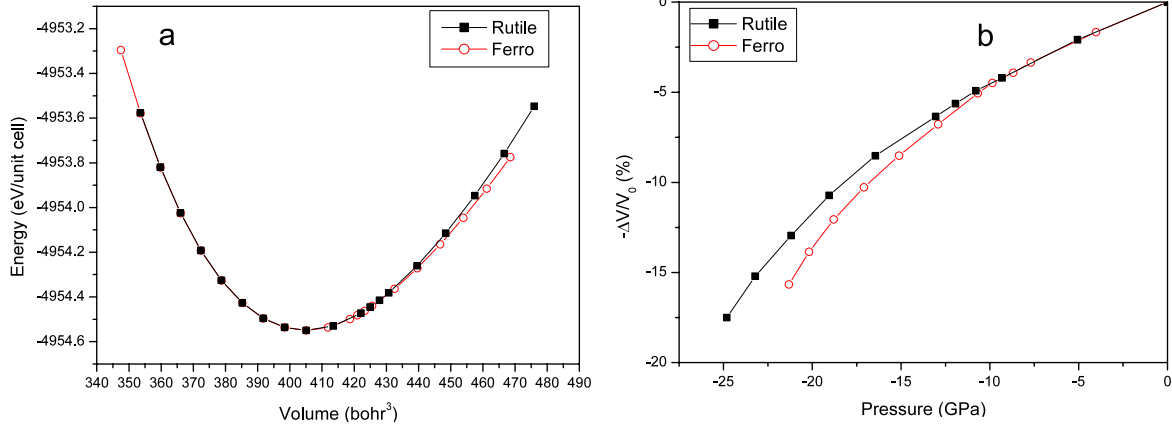


Figure 3. Cohesive energy as a function of unit cell volume (a) and volume expandability as a function of pressure (b) for rutile and ferro TiO<sub>2</sub>.

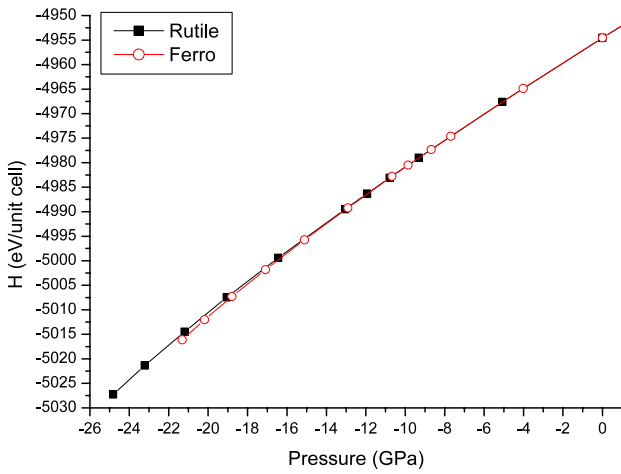


Figure 4. Calculated enthalpy as a function of pressure for rutile and ferro TiO<sub>2</sub>.

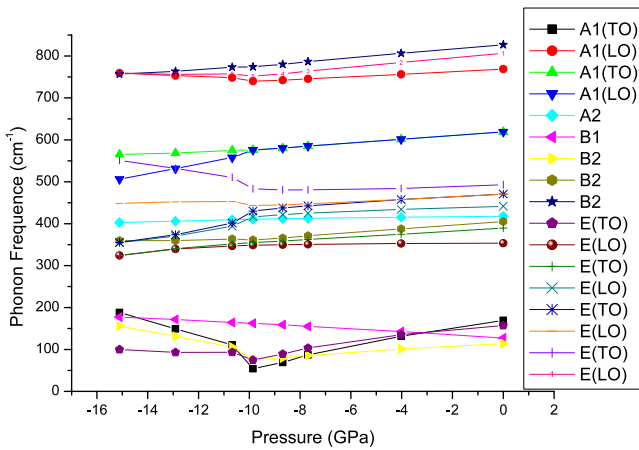


Figure 5. Calculated phonon frequencies at the  $\Gamma$  point of ferro TiO<sub>2</sub> as a function of pressure.

### 3.3. Characteristics of the ferroelectric phase transition

For the purpose of investigating the characteristics of the ferroelectric rutile-to-ferro phase transition in TiO<sub>2</sub>, further calculations have been performed to generate the pressure

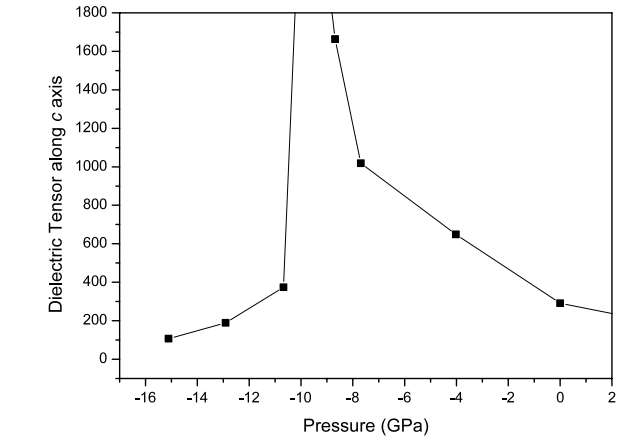


Figure 6. Calculated dielectric tensor along the  $c$  axis in the vicinity of the phase transition.

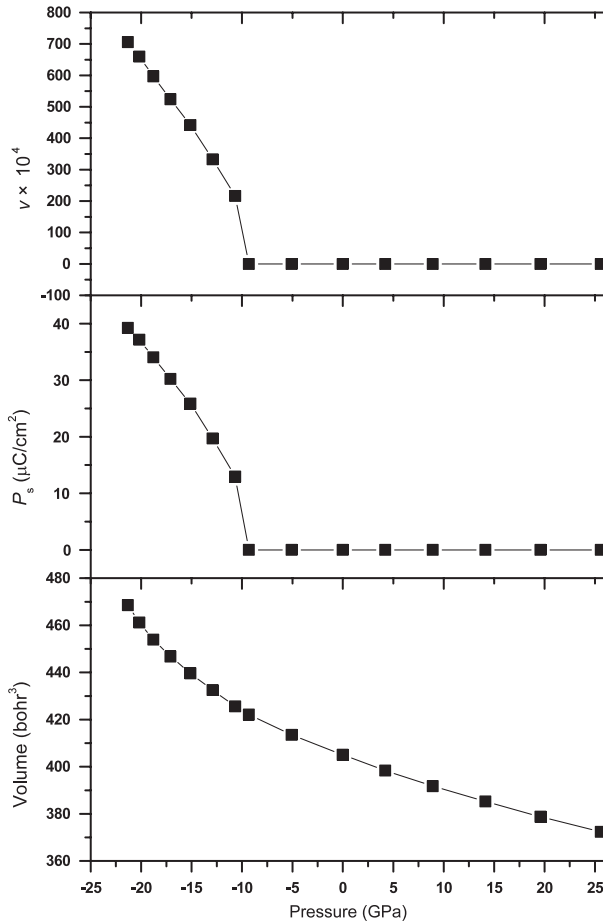
dependence of the unit cell volume, spontaneous polarization  $P_s$  and structural parameter  $v$ , as shown in figure 7. Assuming the crystal is purely ionic by neglecting the electronic polarization,  $P_s$  can be obtained from  $P_s = \sum_i q_i v_i / V$ , where  $q_i$  is the ionic charge of the  $i$ th atom. Although the Born effective charges need to be used to obtain an accurate value of spontaneous polarization, it is acceptable to employ the ionic charges since we focus on the relative values only.

All of the volume,  $P_s$  and  $v$  show continuous decreases towards to zero, with  $P_s$  and  $v$  reaching zero corresponding to the paraelectric rutile TiO<sub>2</sub> at around -10 GPa. This continuous behavior proves a second-order character of the phase transition. Noticeably, the value of  $P_s$  under the pressure -15 GPa is nearly  $20 \mu\text{C cm}^{-2}$  and increases to nearly  $40 \mu\text{C cm}^{-2}$  at -20 GPa. These values are close to the theoretical results for ferroelectric PbTiO<sub>3</sub> calculated in the same way [22], which implies a possibility of negative pressure induced ferro TiO<sub>2</sub> in application.

## 4. Conclusions and discussion

The negative pressure induced rutile-to-ferro phase transition in TiO<sub>2</sub> has been investigated by using the DFT-LDA method.





**Figure 7.** Volume of unit cell,  $P_s$ , and  $v$  as function of pressure.

The three softest phonon modes,  $A_{2u}(\text{TO})$ ,  $E_u(\text{TO})$ , and  $B_{1u}$  of rutile  $\text{TiO}_2$  become negative at  $-10$  GPa,  $-11.5$  GPa, and  $-13$  GPa, respectively, which means that the rutile structure is unstable at  $-10$  GPa and the phase transition is related to the softening behavior of the  $A_{2u}(\text{TO})$  mode as the pressure decreases. The sequential phase  $\text{TiO}_2$  belongs to  $P42nm$  space group and the spontaneous polarization is along the  $c$  axis. The calculated bulk modulus under zero pressure of the *ferro*  $\text{TiO}_2$ , 243.9 GPa, is slightly smaller than the value of rutile  $\text{TiO}_2$ , 250.2 GPa. The values of the structural constants and the phonon frequencies of rutile and the *ferro*  $\text{TiO}_2$  under zero pressure are very close. But under the negative pressure, the *ferro* phase behaves as much softer, consist with the fact that the ferroelectricity favors the more compressible structure. The pressure dependences of unit cell volume, spontaneous polarization  $P_s$ , and structural parameter  $v$  exhibit a continuous decrease toward zero and  $P_s$  and  $v$  do become zero at around  $-10$  GPa, evidencing a second-order character of the phase transition, and the transition pressure is  $-10$  GPa.

It is worthwhile to note that we assume that the ferroelectric phase transition of  $\text{TiO}_2$  is purely displacive instead of order–disorder.  $\text{BaTiO}_3$ , however, which has a similar crystal structure to rutile  $\text{TiO}_2$ , exhibits a complicated case in that it is intermediate between order–disorder and displacive nature. This is because the centro-symmetry of Ti

in the  $\text{TiO}_6$  octahedron is not stable due to the pseudo-Jahn–Teller effects (PJTE) leading into dipole-moment formation in its high temperature phases and being partly ordered at low temperature. This was shown theoretically and experimentally almost 30 years ago [25–27]. Although to the authors’ knowledge no experimental or theoretical investigations have been done to prove that rutile  $\text{TiO}_2$  is completely or partially ordered corresponding to PJTE and the uncertainty of the choice of the purely displacive nature still remains, we believe that rutile  $\text{TiO}_2$  favors the displacive nature because its PJTE is weaker than that in  $\text{BaTiO}_3$ . To elucidate this issue we rewrite equation (5.30) in Bersuker’s book [24] as the following:

$$4F^2/K\Delta - 1 > 0 \quad (3)$$

where  $\Delta$  is the half energy gap between the ground and excited states under consideration, which are mixed by  $T_{1u}$  displacements,  $F$  is the vibronic constant, and  $K$  is the force constant of the  $T_{1u}$  vibrations. The greater the left part, the deeper the adiabatic potential, and the higher the temperature of phase transition to the paraphase, which means stronger PJTE. Considering the series of  $\text{SrTiO}_3$ ,  $\text{TiO}_2$ , and  $\text{BaTiO}_3$ , the Ti–O distance increases from 1.95 to 1.98 Å, then to 2.01 Å. Consequently, the force constant  $K$  and the effective energy distance between the bands decrease while the vibronic constant  $F$  is less influenced. Thus from  $\text{SrTiO}_3$ , to  $\text{TiO}_2$ , then to  $\text{BaTiO}_3$ , the PJTE is strengthened and the conditions of ferroelectricity occurrence are improved. This trend is consistent with the fact that both  $\text{SrTiO}_3$  and rutile  $\text{TiO}_2$  are incipient ferroelectrics but  $\text{BaTiO}_3$  is a true one. For weak PJTE the ground state is softened but remains stable, which implies there is no (local) dipolar moment in the paraphase and the transition is not order–disorder but displacive. Furthermore, the PJTE in rutile  $\text{TiO}_2$  is much stronger than that in  $\text{SrTiO}_3$ . Since in-plane tensile strains can change  $\text{SrTiO}_3$  to true ferroelectricity in epitaxial films [23], less tensile strain will act on  $\text{TiO}_2$ . This adds further validity to the present theoretical investigations.

### Acknowledgment

The authors would like to thank Zhejiang University National Science Park for the technical support provided during the period of this research.

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